Complete description of polarization effects in the nonlinear Compton scattering
II. Linearly polarized laser photons

D.Yu. Ivanov[1)], G.L. Kotkin[2)], V.G. Serbo[2)]

[1)]*Sobolev Institute of Mathematics, Novosibirsk, 630090, Russia*

[2)]*Novosibisk State University, Novosibirsk, 630090, Russia*

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Abstract

We consider emission of a photon by an electron in the field of a strong laser wave. Polarization effects in this process are important for a number of physical problems. We discuss a probability of this process for linearly polarized laser photons and for arbitrary polarization of all other particles. We obtain the complete set of functions which describe such a probability in a compact form.

1 Introduction

Let us consider emission of a photon by an electron in the field of a strong laser wave. The complete description of polarization effects in the case of the circularly polarized laser wave was considered in our paper [1]. The present paper is a continuation of Ref. [1] and we use the same notations[1]. Thus, we deal with the process of nonlinear Compton scattering

$$e(q) + n\,\gamma_L(k) \to e(q') + \gamma(k')\,, \tag{1}$$

when the electromagnetic laser field is described by 4-potential

$$A^\mu(x) = A^\mu\,\cos(kx), \tag{2}$$

where A^μ is the amplitude of this field. Such a process with absorbtion of $n = 1,\,2,\,3,\,4$ linearly polarized laser photons was observed in recent experiment at SLAC [2].

The method of calculation for such process was developed by Nikishov and Ritus [3]. It is based on the exact solution of the Dirac equation in the external electromagnetic plane

[1]Below we shall quote formulas from paper [1] by a double numbering, for example, Eq. (1.21) means Eq. (21) from Ref. [1].

wave. Some particular polarization properties of this process for the linearly polarized laser photons were considered in [3, 4]. In the present paper we give the complete description of the nonlinear Compton scattering for the case of linearly polarized laser photons and arbitrary polarization of all other particles. We follow the method of Nikishov and Ritus in the form presented in [5] §101.

In the next section we briefly describe the kinematics. The cross section, including polarization of all particles, is obtained in Sect. 3. In Sect. 4 the polarization of final particles, averaged over azimuthal angle, is obtained. In Sect. 5 we summarize our results and compare them with those known in the literature. In Appendix we give a comparison of the obtained cross section in the limit of weak laser field with the known results for the linear Compton scattering.

2 Kinematics

All kinematical relations derived in Sect. 2 of Ref. [1] are valid for the considered case as well. In particular, the parameter of nonlinearity is defined via the mean value of squared 4-potential in the same form[2]

$$\xi = \frac{e}{mc^2}\sqrt{-\langle A_\mu(x) A^\mu(x)\rangle} = \frac{e}{mc^2}\sqrt{-\tfrac{1}{2}\, A_\mu A^\mu}\,, \tag{3}$$

where e and m is the electron charge and mass, c is the velocity of light. Therefore, the amplitude of 4-potential can be presented in the form

$$A^\mu = \frac{\sqrt{2}\, mc^2}{e}\, \xi\, e_L^\mu\,, \quad e_L e_L = -1\,, \tag{4}$$

where e_L^μ is the unit 4-vector describing the polarization of the laser photons.

In the frame of reference, in which the electron momentum $\mathbf{p}$ is anti-parallel to the initial photon momentum $\mathbf{k}$ (we call it as a "collider system"), we direct the z-axis along the initial electron momentum $\mathbf{p}$. For the discussed problem it is convenient to choose the x-axis along the direction of the laser linear polarization, i.e. along the vector $\mathbf{e}_L$. Azimuthal angles φ, β and β' of vectors $\mathbf{k}'$ and electron polarizations $\boldsymbol{\zeta}$ and $\boldsymbol{\zeta}'$ are defined with respect to this x-axis. After that, the unit 4-vector e_L can be presented in the form

$$e_L = e^{(1)}\, \sin\varphi - e^{(2)}\, \cos\varphi\,, \tag{5}$$

where the unit 4-vectors $e^{(1)}$ and $e^{(2)}$ are given in (1.19). This equation determines the quantities $\sin\varphi$ and $\cos\varphi$ in an invariant form, suitable for any frame of reference. As in paper [1], we use the Stokes parameters ξ_i and ξ_i' to describe the polarization of the initial photon and the detector polarization of the final photon, respectively. They are defined

[2]Note, that our definition of this parameter differs from that used in Refs. [3] by additional factor $1/\sqrt{2}$.

with respect to the $x'y'z'$-axes which are fixed to the scattering plane. The x'-axis is the same for both photons and perpendicular to the scattering plane:

$$x' \ \parallel \ \mathbf{k} \times \mathbf{k}'; \tag{6}$$

the y'-axes are in that plane, in particular,

$$y' \ \parallel \ \mathbf{k} \times (\mathbf{k} \times \mathbf{k}') = -\omega^2 \mathbf{k}'_\perp \tag{7}$$

for the initial photon and

$$y' \ \parallel \ \mathbf{k}' \times (\mathbf{k} \times \mathbf{k}') \tag{8}$$

for the final photon. The azimuthal angle of the linear polarization of the laser photon in the collider system equals zero with respect to the xyz-axes, and it is $\varphi - (\pi/2)$ with respect to the $x'y'z'$-axes. Therefore, in the considered case of 100 % linearly polarized laser beam one has

$$\xi_1 = -\sin 2\varphi, \quad \xi_2 = 0, \quad \xi_3 = -\cos 2\varphi\,. \tag{9}$$

At small emission angles of the final photon $\theta_\gamma \ll 1$, the final photon moves almost along the direction of the z-axis and $\mathbf{k} \times \mathbf{k}'$ azimuth is approximately equal to $\varphi - (\pi/2)$. Let $\check{\xi}'_j$ be the detected Stokes parameters for the final photons, fixed to the xyz-axes. They are connected with ξ'_j by the relations

$$\xi'_1 \approx -\check{\xi}'_1 \cos 2\varphi + \check{\xi}'_3 \sin 2\varphi\,, \ \ \xi'_2 = \check{\xi}'_2\,, \ \ \xi'_3 \approx -\check{\xi}'_3 \cos 2\varphi - \check{\xi}'_1 \sin 2\varphi\,. \tag{10}$$

The polarization properties of the initial and final electrons are described in the same form as in paper [1].

3 The effective cross section

As in paper [1], we present the effective differential cross section in the form[3]

$$d\sigma(\xi'_i, \zeta'_j) = \frac{r_e^2}{4x} \sum_n F^{(n)}\, d\Gamma_n\,, \quad d\Gamma_n = \delta(q + n\,k - q - k')\, \frac{d^3k'}{\omega'} \frac{d^3q'}{q'_0}\,, \tag{11}$$

where $r_e = \alpha/m$ is the classical electron radius, and

$$F^{(n)} = F_0^{(n)} + \sum_{j=1}^{3} \left(F_j^{(n)} \xi'_j \ + \ G_j^{(n)} \zeta'_j \right) + \sum_{i,j=1}^{3} H_{ij}^{(n)}\, \zeta'_i\, \xi'_j\,. \tag{12}$$

We have calculated functions $F_j^{(n)}$, $G_j^{(n)}$ and $H_{ij}^{(n)}$ using the standard technic presented in [5] §101. All the necessary traces have been calculated using the package MATEMATIKA. In the considered case almost all dependence on the nonlinearity parameter ξ

[3]Below we use the system of units in which $c = 1$, $\hbar = 1$.

accumulates in three functions:

$$\begin{aligned}
\tilde{f}_n &= 4\,[A_1(n,\,a,\,b)]^2 - 4A_0(n,\,a,\,b)A_2(n,\,a,\,b)\,, \\
\tilde{g}_n &= \frac{4n^2}{z_n^2}\,[A_0(n,\,a,\,b)]^2\,, \\
\tilde{h}_n &= \frac{4n}{a}\,A_0(n,\,a,\,b)\,A_1(n,\,a,\,b)\,,
\end{aligned} \tag{13}$$

where functions $A_k(n,\,a,\,b)$ were introduced in [3] as follows

$$A_k(n,\,a,\,b) = \int\limits_{-\pi}^{\pi} \cos^k\psi\;\exp\left[\,\mathrm{i}\,(n\psi - a\sin\psi + b\sin 2\psi)\right]\frac{d\psi}{2\pi}\,. \tag{14}$$

The arguments of these functions are

$$a = e\left(\frac{Ap}{kp} - \frac{Ap'}{kp'}\right)\,,\quad b = \frac{1}{8}\,e^2A^2\left(\frac{1}{kp} - \frac{1}{kp'}\right) \tag{15}$$

or in more convenient variables they are

$$a = \sqrt{2}\,\xi\,m\left(\frac{e_Lp}{kp} - \frac{e_Lp'}{kp'}\right)\,,\quad b = \frac{y}{2(1-y)x}\,\xi^2\,. \tag{16}$$

In the collider system one has

$$a = -z_n\,\sqrt{2}\,\cos\varphi\,, \tag{17}$$

where

$$z_n = \frac{\xi}{\sqrt{1+\xi^2}}\,n\,s_n$$

was defined in (1.41). Among the functions $A_0(n,\,a,\,b)$, $A_1(n,\,a,\,b)$ and $A_2(n,\,a,\,b)$ there is a useful relation

$$(n-2b)\,A_0(n,\,a,\,b) - a\,A_1(n,\,a,\,b) + 4b\,A_2(n,\,a,\,b) = 0\,. \tag{18}$$

To find the photon spectrum, one needs also functions (13) averaged over the azimuthal angle φ:

$$\langle\tilde{f}_n\rangle = \int_0^{2\pi}\tilde{f}_n\,\frac{d\varphi}{2\pi}\,,\quad \langle\tilde{g}_n\rangle = \int_0^{2\pi}\tilde{g}_n\,\frac{d\varphi}{2\pi}\,. \tag{19}$$

For small values of $\xi\to 0$ or $y\to 0$, we have

$$\begin{aligned}
a &\propto \sqrt{y\,\xi^2}\,,\;\; b\propto y\,\xi^2\,, \\
A_{0,\,2}(n,\,a,\,b) &\propto \left(y\,\xi^2\right)^{n/2}\,,\;\; A_1(n,\,a,\,b)\propto\left(y\,\xi^2\right)^{(n-1)/2}\,, \\
\tilde{f}_n,\,\tilde{g}_n,\,\tilde{h}_n &\propto \left(y\,\xi^2\right)^{n-1}\,,
\end{aligned} \tag{20}$$

in particular, at $\xi = 0$ or at $y = 0$

$$\tilde{f}_1 = \langle \tilde{f}_1 \rangle = \langle \tilde{g}_1 \rangle = \tilde{h}_1 = 1\,, \quad \tilde{g}_1 = 1 + \cos 2\varphi\,. \tag{21}$$

The results of our calculations are the following. First, we define the auxiliary functions

$$\begin{aligned}
X_n &= \tilde{f}_n - (1 + c_n) \Big[(1 - \Delta\, r_n)\, \tilde{g}_n - \tilde{h}_n\, \cos 2\varphi \Big]\,, \\
Y_n &= (1 + c_n)\tilde{g}_n - 2\tilde{h}_n \cos^2\varphi\,, \\
V_n &= \tilde{f}_n\, \cos 2\varphi + 2(1 + c_n) \Big[(1 - \Delta\, r_n)\, \tilde{g}_n\, - 2\tilde{h}_n\, \cos^2\varphi \Big] \sin^2\varphi\,,
\end{aligned} \tag{22}$$

where we use the notation

$$\Delta = \frac{\xi^2}{1 + \xi^2}\,,$$

c_n and r_n is defined in (1.10) and (1.11), respectively.

The item $F_0^{(n)}$, related to the total cross section (1.35), reads

$$F_0^{(n)} = \frac{2 - 2y + y^2}{1 - y}\, \tilde{f}_n - \frac{s_n^2}{1 + \xi^2}\, \tilde{g}_n\,. \tag{23}$$

The polarization of the final photons $\xi_j^{(f)}$ is given by Eq. (1.36) where

$$\begin{aligned}
F_1^{(n)} &= 2\, X_n\, \sin 2\varphi\,, \quad F_3^{(n)} = -2\, V_n + \frac{s_n^2}{1 + \xi^2}\, \tilde{g}_n\,, \\
F_2^{(n)} &= \frac{y s_n}{\sqrt{1 + \xi^2}} \Big[\tilde{h}_n\, \zeta_1\, \sin 2\varphi - Y_n\, \zeta_2 \Big] + \left(\frac{2 - y}{1 - y}\, \tilde{f}_n - \frac{s_n^2}{1 + \xi^2}\, \tilde{g}_n \right) y\, \zeta_3\,,
\end{aligned} \tag{24}$$

The polarization of the final electrons $\zeta_j^{(f)}$ is given by Eqs. (1.37), (1.38) with

$$\begin{aligned}
G_1^{(n)} &= \left(2\tilde{f}_n - \frac{s_n^2}{1 + \xi^2}\, \tilde{g}_n \right) \zeta_1 + \frac{y s_n}{(1 - y)\sqrt{1 + \xi^2}}\, \tilde{h}_n\, \zeta_3\, \sin 2\varphi\,, \\
G_2^{(n)} &= \left(2\tilde{f}_n - \frac{s_n^2}{1 + \xi^2}\, \tilde{g}_n \right) \zeta_2 - \frac{y s_n}{(1 - y)\sqrt{1 + \xi^2}}\, Y_n \zeta_3\,, \\
G_3^{(n)} &= -\frac{y s_n}{\sqrt{1 + \xi^2}} \left(\tilde{h}_n\, \zeta_1\, \sin 2\varphi - Y_n \zeta_2 \right) \\
&\quad + \left(\frac{2 - 2y + y^2}{1 - y} \tilde{f}_n - \frac{1 - y + y^2}{1 - y} \frac{s_n^2}{1 + \xi^2}\, \tilde{g}_n \right) \zeta_3\,.
\end{aligned} \tag{25}$$

At last, the correlation of the final particles' polarizations are

$$\begin{aligned}
H_{11}^{(n)} &= \frac{2 - 2y + y^2}{1 - y}\, X_n \zeta_1\, \sin 2\varphi - y \left(\frac{2 - y}{1 - y}\, V_n - \frac{s_n^2}{1 + \xi^2}\, \tilde{g}_n \right) \zeta_2 - \frac{y s_n}{\sqrt{1 + \xi^2}} Y_n \zeta_3\,, \\
H_{21}^{(n)} &= \frac{y}{1 - y} \left[(2 - y) V_n - \frac{s_n^2}{1 + \xi^2}\, \tilde{g}_n \right] \zeta_1 + \frac{2 - 2y + y^2}{1 - y} X_n \zeta_2\, \sin 2\varphi
\end{aligned}$$

$$
\begin{aligned}
& +\frac{y s_n}{\sqrt{1+\xi^2}}\tilde{h}_n\,\zeta_3\,\sin 2\varphi\,, \\
H_{31}^{(n)} &= \frac{y s_n}{(1-y)\sqrt{1+\xi^2}}\Big(Y_n\zeta_1-\tilde{h}_n\,\zeta_2\,\sin 2\varphi\Big)+2X_n\zeta_3\,\sin 2\varphi\,, \\
H_{12}^{(n)} &= \frac{y s_n}{(1-y)\sqrt{1+\xi^2}}\,\tilde{h}_n\,\sin 2\varphi\,, \quad H_{22}^{(n)}=-\frac{y s_n}{(1-y)\sqrt{1+\xi^2}}\,Y_n\,, \\
H_{32}^{(n)} &= \frac{y}{1-y}\left[(2-y)\tilde{f}_n-\frac{s_n^2}{1+\xi^2}\,\tilde{g}_n\right], \\
H_{13}^{(n)} &= -\left(\frac{2-2y+y^2}{1-y}V_n-\frac{s_n^2}{1+\xi^2}\,\tilde{g}_n\right)\zeta_1-\frac{y(2-y)}{1-y}\,X_n\zeta_2\,\sin 2\varphi \\
& +\frac{y s_n}{\sqrt{1+\xi^2}}\,\tilde{h}_n\,\zeta_3\,\sin 2\varphi\,, \\
H_{23}^{(n)} &= \frac{y(2-y)}{1-y}\,X_n\zeta_1\,\sin 2\varphi-\left(\frac{2-2y+y^2}{1-y}V_n-\frac{1-y+y^2}{1-y}\frac{s_n^2}{1+\xi^2}\,\tilde{g}_n\right)\zeta_2 \\
& +\frac{y s_n}{\sqrt{1+\xi^2}}Y_n\zeta_3\,, \\
H_{33}^{(n)} &= -\frac{y s_n}{(1-y)\sqrt{1+\xi^2}}\Big(\tilde{h}_n\,\zeta_1\,\sin 2\varphi+Y_n\zeta_2\Big)-\left(2\,V_n-\frac{s_n^2}{1+\xi^2}\,\tilde{g}_n\right)\zeta_3\,.
\end{aligned}
\tag{26}
$$

4 Averaged polarization of the final particles

In many application it is important to know the averaged over azimuthal angle φ polarization of the final photon and electron in the collider system. To find it, we can use the same method as for the linear Compton scattering (see Refs. [9], [7] and [8]). We substitute ξ_j, ζ_j, ξ'_j and ζ'_j from Eqs. (9), (1.51), (10) and (1.56), respectively, into Eq. (12) and obtain after integration over azimuthal angle φ:

$$
\begin{aligned}
\frac{d\sigma_n(\xi'_i,\zeta'_j)}{dy} \approx \frac{\pi r_e^2}{2x}\,\Big(&\langle F_0^{(n)}\rangle+\langle\Phi_1^{(n)}\rangle\,\check{\xi}'_1+\langle F_2^{(n)}\rangle\,\check{\xi}'_2+\langle\Phi_3^{(n)}\rangle\,\check{\xi}'_3+ \\
&+G_\perp^{(n)}\boldsymbol{\zeta}_\perp\boldsymbol{\zeta}'_\perp+G_\parallel^{(n)}\zeta_3\zeta'_3+\sum_{i,j=1}^{3}\langle H_{ij}^{(n)}\,\zeta'_i\,\xi'_j\rangle\Big)\,,
\end{aligned}
\tag{27}
$$

where

$$
\begin{aligned}
\langle F_0^{(n)}\rangle &= \left(\frac{1}{1-y}+1-y\right)\langle\tilde{f}_n\rangle-\frac{s_n^2}{1+\xi^2}\,\langle\tilde{g}_n\rangle\,, \\
\langle\Phi_1^{(n)}\rangle &= \langle -F_1^{(n)}\,\cos 2\varphi-F_3^{(n)}\,\sin 2\varphi\rangle=0\,, \\
\langle F_2^{(n)}\rangle &= \left(\frac{2-y}{1-y}\,\langle\tilde{f}_n\rangle-\frac{s_n^2}{1+\xi^2}\,\langle\tilde{g}_n\rangle\right)y\,\zeta_3\,, \\
\langle\Phi_3^{(n)}\rangle &= \langle F_1^{(n)}\,\sin 2\varphi-F_3^{(n)}\,\cos 2\varphi\rangle\,, \\
G_\perp^{(n)} &= 2\langle\tilde{f}_n\rangle-\frac{s_n^2}{1+\xi^2}\,\langle\tilde{g}_n\rangle\,,
\end{aligned}
$$

$$G_{\parallel}^{(n)} = \frac{2-2y+y^2}{1-y}\langle \tilde{f}_n\rangle - \frac{1-y+y^2}{1-y}\frac{s_n^2}{1+\xi^2}\langle \tilde{g}_n\rangle\,,$$

As a result, the averaged Stokes parameters of the final photon are

$$\langle \breve{\xi}_{1(n)}^{(f)}\rangle \approx 0\,,\quad \langle \breve{\xi}_{2(n)}^{(f)}\rangle \approx \frac{\langle F_2^{(n)}\rangle}{\langle F_0^{(n)}\rangle}\,,\quad \langle \breve{\xi}_{3(n)}^{(f)}\rangle \approx \frac{\langle \Phi_3^{(n)}\rangle}{\langle F_0^{(n)}\rangle}\,, \tag{28}$$

and the averaged polarization of the final electron is

$$\langle \boldsymbol{\zeta}_{\perp(n)}^{(f)}\rangle \approx \frac{G_{\perp}^{(n)}}{\langle F_0^{(n)}\rangle}\,\boldsymbol{\zeta}_{\perp}\,,\quad \langle \zeta_{3(n)}^{(f)}\rangle \approx \frac{G_{\parallel}^{(n)}}{\langle F_0^{(n)}\rangle}\,\zeta_3\,. \tag{29}$$

Note that averaged Stokes parameters of the final photon do not depend on $\boldsymbol{\zeta}_{\perp}$ and that averaged polarization vector of the final electron is not equal zero only if $\boldsymbol{\zeta} \neq 0$. These properties are similar to those in the linear Compton scattering.

5 Summary and comparison with other papers

Our main result is given by Eqs. (23)–(26) which present 16 functions $F_0,\ F_j,\ G_j$ and H_{ij} with $i,\ j = 1 \div 3$. They describe completely all polarization properties of the nonlinear Compton scattering in a rather compact form.

In the literature we found 4 functions which can be compared with ours $F_0, F_1,\ F_2,\ F_3$. They enter the total cross section (1.35), (1.54), the differential cross sections (1.53), (1.54) and the Stokes parameters of the final photons (1.36). The function F_0 was calculated in [3], the functions F_j were obtained in [4]. Our results (23), (24) coincide with the above mentioned ones.

The polarization of the final electrons is described by functions G_j (25). They enter the polarization vector $\boldsymbol{\zeta}^{(f)}$ given by exact (1.38), (1.61) and approximate (1.67) equations. The correlation of the final particles' polarizations are described by functions H_{ij} given in Eqs. (26). We did not find in the literature any on these functions.

At small ξ^2 all harmonics with $n > 1$ disappear due to properties (20) and (21),

$$d\sigma_n(\xi_i', \zeta_j') \propto \xi^{2(n-1)} \quad \text{at} \quad \xi^2 \to 0\,. \tag{30}$$

We checked that in this limit our expression for $d\sigma(\xi_i', \zeta_j')$ coincides with the result known for the linear Compton effect, see Appendix.

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Appendix: Limit of the weak laser field

At $\xi^2 \to 0$, the cross section (11) has the form

$$d\sigma(\xi_i', \zeta_j') = \frac{r_e^2}{4x}\, F\, d\Gamma; \quad d\Gamma = \delta(p+k-p-k')\, \frac{d^3k'}{\omega'}\frac{d^3p'}{E'}\,, \tag{31}$$

where

$$F = F_0 + \sum_{j=1}^{3}\left(F_j\xi_j' \;+\; G_j\zeta_j'\right) + \sum_{i,j=1}^{3} H_{ij}\,\zeta_i'\,\xi_j'\,. \tag{32}$$

To compare it with the cross section for the linear Compton scattering, we should take into account that the Stokes parameters of the initial photon have values (9) and that our invariants c_1, s_1, r_1 and auxiliary functions (22) transforms at $\xi^2 = 0$ to

$$\begin{aligned} c_1 &\;\to\; c = 1-2r\,, \quad s_1 \to s = 2\sqrt{r(1-r)}\,, \quad r_1 \to r = \frac{y}{x(1-y)}\,, \\ X_1 &\;\to\; -c\,, \quad Y_1 \to c(1-\xi_3)\,, \quad V_1 \to -\xi_3\,. \end{aligned} \tag{33}$$

Our functions

$$\begin{aligned} F_0 &= \frac{1}{1-y} + 1 - y - s^2(1-\xi_3)\,, \quad F_1 = 2c\xi_1\,, \quad F_3 = s^2 + (1+c^2)\,\xi_3\,, \\ F_2 &= -ysc\,\zeta_2 + y\left(\frac{1}{1-y} + c^2\right)\zeta_3 - ys\,\zeta_1\xi_1 + ys\,(c\zeta_2 + s\zeta_3)\;\xi_3 \end{aligned} \tag{34}$$

coincide with those in [7]. Our functions

$$\begin{aligned} G_1 &= (1+c^2+s^2\xi_3)\zeta_1 - \frac{y\,s}{1-y}\xi_1\zeta_3\,, \\ G_2 &= (1+c^2+s^2\xi_3)\;\zeta_2 - \frac{ysc}{1-y}\,(1-\xi_3)\;\zeta_3\,, \\ G_3 &= ys\xi_1\zeta_1 + ysc(1-\xi_3)\;\zeta_2 + \left[1 + \left(\frac{1}{1-y} - y\right)\;(c^2+s^2\xi_3)\right]\;\zeta_3. \end{aligned} \tag{35}$$

coincide with functions Φ_j given by Eqs. (31) in [8]. At last, we check that our functions

$$\begin{aligned} H_{11} &= \frac{2-2y+y^2}{1-y}\,c\,\zeta_1\,\xi_1 + y\left[\frac{2-y}{1-y}\,\xi_3 + s^2(1-\xi_3)\right]\;\zeta_2 - ysc(1-\xi_3)\,\zeta_3\,, \\ H_{21} &= -\frac{y}{1-y}\left[s^2 + (1-y+c^2)\xi_3\right]\zeta_1 + \frac{2-2y+y^2}{1-y}\,c\xi_1\zeta_2 - ys\xi_1\zeta_3\,, \\ H_{31} &= \frac{ycs}{1-y}\,(1-\xi_3)\zeta_1 + \frac{ys}{1-y}\,\xi_1\zeta_2 + 2c\xi_1\zeta_3\,, \quad H_{12} = -\frac{ys}{1-y}\,\xi_1\,, \\ H_{22} &= -\frac{ycs}{1-y}\,(1-\xi_3)\,, \quad H_{32} = \frac{y}{1-y}\left[2-y-s^2(1-\xi_3)\right]\,, \\ H_{13} &= s^2\,\zeta_1 + \left(1+c^2+\frac{y^2}{1-y}\right)\;\xi_3\zeta_1 - yc\,\frac{2-y}{1-y}\,\xi_1\,\zeta_2 - ys\,\xi_1\zeta_3\,, \end{aligned} \tag{36}$$

$$H_{23} = y\frac{2-y}{1-y}\,c\,\xi_1\,\zeta_1 + \frac{1-y+y^2}{1-y}\,s^2\,\zeta_2 + \left(1+c^2+\frac{y^2c^2}{1-y}\right)\xi_3\zeta_2 + ysc(1-\xi_3)\,\zeta_3\,,$$
$$H_{33} = \frac{ys}{1-y}\,\xi_1\,\zeta_1 - \frac{ysc}{1-y}\,(1-\xi_3)\,\zeta_2 + s^2\,\zeta_3 + (1+c^2)\,\xi_3\,\zeta_3$$

coincide with the corresponding functions from [6]. At such a comparison, one needs to take into account that the set of unit 4-vectors, used in our paper (see Eqs. (1.19), (1.23)), and one used in paper [6] are different. The relations between our notations and the notations used in [6] are given by Eqs. (1.75).

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